DYNAMICS OF A TEST PARTICLE UNDER DIRECT AND REFLECTED RADIATIONS IN THE COLLINEAR RESTRICTED FOUR-BODY PROBLEM

SHORT SYNOPSIS

Submitted in partial fulfillment of the requirement of the degree of

DOCTOR OF PHILOSOPHY

by

Usha

Registration No.- MRUMADN012K210082

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MANAV RACHNA UNIVERSITY

2026

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DECLARATION BY THE SCHOLAR

I hereby declare that the short synopsis entitled **Dynamics of a test particle under direct and reflected radiations in the collinear restricted four-body problem** being submitted in partial fulfillment of the requirement for the degree of **Doctor of Philosophy** in the Department of Sciences (Mathematics) under the School of Sciences of Manav Rachna University, Faridabad during the academic year 2025-2026.

It is the work that is expected to be carried out under the supervision of **Dr. Mohammad Javed Idrisi** and co-supervision of **Dr. Ruchi Gupta**.

Further, it is declared that the short synopsis is within the plagiarism limit and a copy of the same is attached as per the University norms.

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CERTIFICATE BY SUPERVISOR(S)

This is to certify that, the short synopsis entitled **Dynamics of a test particle under direct and reflected radiations in the collinear restricted four-body problem** is an original piece of work done by research scholar **Usha** under the supervision of **Dr. Mohammad Javed Idrisi** and co-supervision of **Dr. Ruchi Gupta** for the degree of **Doctor of Philosophy** in the Department of Sciences (Mathematics) under the School of Sciences of Manav Rachna University, Faridabad during the academic year 2025-2026.

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ABSTRACT

In celestial mechanics, R4BP plays a fundamental role because it explains how an infinitesimal mass particle moves under the force of gravity of three primaries which are very massive. The realism of dynamical models used to study planetary systems, motion of dust particles and spacecraft trajectories has significantly increased in recent years with the addition of non-gravitational forces like radiation pressure. The current research examines the study of the motion of a particle whose mass is infinitesimal in the collinear restricted four-body problem (CR4BP) under direct and reflected radiations effect. The three bodies which are massive in the proposed study will be supposed to move in orbits that are circular while retaining a collinear configuration. The infinitesimal mass particle moves with the gravitational attraction of the massive primary bodies together with the effects of direct radiation pressure and reflected radiation (albedo effect). Radiation factors will be added to the classical restricted four-body problem for formulating the equations of motion of the infinitesimal mass particle. Both analytical and numerical approaches will be used to examine how radiation variables affect the position and stability of equilibrium points, and by using graphical and numerical methods we will discuss dynamical behavior of system. The findings of this study are going to lead to a better understanding of particle dynamics in radiation-dominated celestial systems, with possible implications in astrophysical science, planetary dynamics, and space mission design.

Keywords: Restricted Four-Body Problem (R4BP), Collinear Configuration, Direct Radiation Pressure, Reflected Radiation (Albedo Effect), Equilibrium Points and Stability

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List of Abbreviations

R3BP	Restricted Three-Body Problem
CR3BP	Circular Restricted Three-Body Problem
R4BP	Restricted Four-Body problem
CR4BP	Collinear Restricted Four-Body Problem
PCR4BP	Photo-gravitational Collinear Restricted Four-Body Problem
EPs	Equilibrium Points
EoM	Equations of Motion
RoM	Regions of Motion

1. Introduction

1.1 Restricted Three-Body Problem

The Restricted Three-Body problem studies the motion of a particle with infinitesimal mass under the force of gravity of two heavy primaries, and the infinitesimal mass particle does not affect the motion of the heavy primaries. If the heavy primaries move in orbits, which is circular around their common center of mass, then it is the circular restricted three-body problem.

Assumptions for the CR3BP

- The distance between the primaries is equal to unity.
- The total mass of the massive primaries is also equal to unity.
- The axes of the primaries, which are principal axes, must be parallel with synodic axes throughout the motion.
- The equatorial axis must be in the plane of motion defined by the primaries.

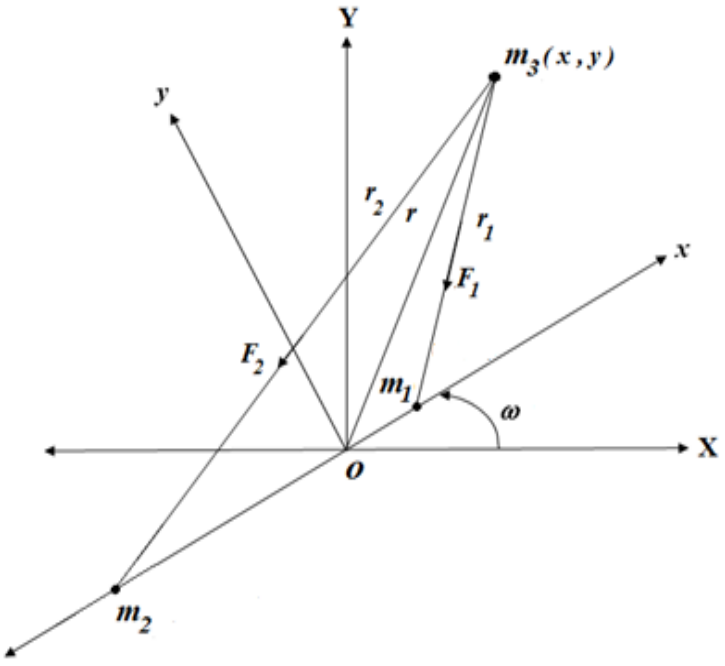


Fig. 1: Circular Restricted Three-Body problem

The EoM of a test particle with mass m are given by

$$\boldsymbol{ma = F = F_1 + F_2} \tag{1}$$

where,

$\boldsymbol{F}$ = Total force acting on m.

$\boldsymbol{F_1}$ = Gravitational force on m due to m_1.

$\boldsymbol{F_2}$ = Gravitational force on m due to m_2.

On simplifying equation (1), we got the equations of motion

$$\left.\begin{aligned} \ddot{x} - 2\dot{y} &= x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu(x+1-\mu)}{r_2^3} \\ \ddot{y} + 2\dot{x} &= y\left[1 - \frac{(1-\mu)}{r_1^3} - \frac{\mu}{r_2^3}\right] \end{aligned}\right\} \tag{2}$$

We can define a function U known as potential function,

$$U(x,y) = \frac{x^2+y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}. \tag{3}$$

Therefore, we have

$$\left.\begin{aligned} \ddot{x} - 2\dot{y} &= U_x \\ \ddot{y} + 2\dot{x} &= U_y \end{aligned}\right\} \tag{4}$$

Where U_x and U_y are the first-order partial derivatives of potential function U.

Equilibrium Points (EPs) / Libration Points (LPs)

The equilibrium points are obtained by taking first-order derivatives w.r.t x and y of the potential function equal to zero.

For the collinear configuration:

- The EPs lie on the *x*-axis ($y = 0$), are called collinear equilibrium points.
- The EPs lie on the *xy*-plane ($x \neq$ and $y \neq 0$), are called non-collinear equilibrium points.
- By the distribution of mass and configuration of the primaries we can find the number of Eps.

Stability Analysis

To analyze stability, small perturbations are introduced around equilibrium points:

$$x = x_0 + \xi,\ y = y_0 + \eta$$

Linearizing the EoM leads to a system of variational equations:

$$\left.\begin{aligned} \ddot{\xi} - 2\omega\dot{\eta} &= \xi \overset{o}{U}_{xx} + \eta \overset{o}{U}_{xy} \\ \ddot{\eta} + 2\omega\dot{\xi} &= \xi \overset{o}{U}_{yx} + \eta \overset{o}{U}_{yy} \end{aligned}\right\} \tag{5}$$

The following is the characteristic equation associated with the system given in equations (5)

$$\lambda^4 + \left(4\omega^2 - \overset{o}{U}_{xx} - \overset{o}{U}_{yy}\right)\lambda^2 + \left(\overset{o}{U}_{xx}\overset{o}{U}_{yy} - \overset{o}{U}_{xy}^2\right) = 0. \tag{6}$$

The characteristic Eqn. (6) derived from this system determines the nature of eigenvalues:

- **Pure imaginary eigenvalues** → Stable motion
- **Real eigenvalues** → Instability
- **Complex with real part** → Saddle-type instability

In most cases, unstable points are collinear points while the non-collinear are stable under some conditions.

1.2 R4BP

The motion of a particle whose mass is infinitesimal under the gravitational pull of various massive bodies is described by the restricted multi-body problem. In the R4BP, three heavy primaries affect the test particle's motion which is an extended version of the R3BP whereas the primaries in the

CR4BP lie along a straight line, those in the R4BP around their shared center of mass move in orbit, which is circular. These configurations make it possible to examine EPs and the particle's dynamic behavior in multi-body systems.

Assumptions for the formulation of CR4BP

- The three primaries, with masses m_1, m_2, m_3 are arranged collinearly on a straight line.
- The primary's motion is governed by Newtonian gravitational law, and they form a known solution of the three-body problem (typically a rotating configuration).
- The fourth body has negligible mass and does not influence the dynamics of the primaries.
- The system is analyzed in a rotating reference frame where the primaries remain fixed on the x-axis.
- Units are normalized such that:
  - Total mass $m_1 + m_2 + m_3 = 1$.
  - Distance between primaries is unity.
  - Gravitational constant $G = 1$.

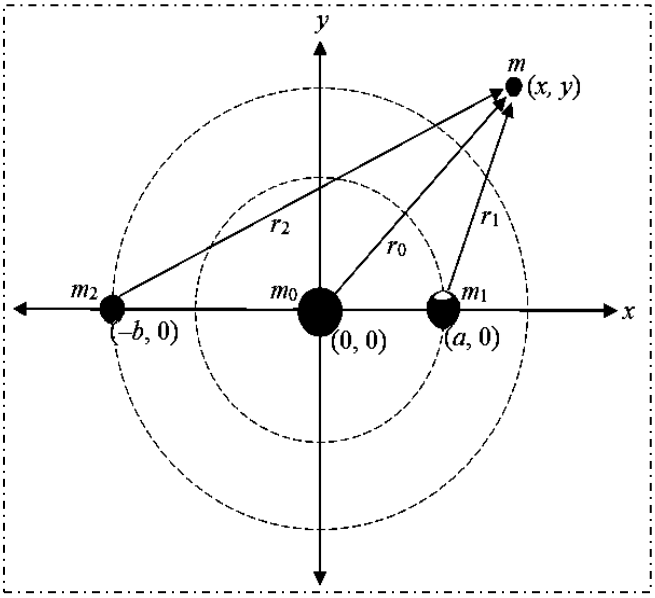


Fig. 2: Collinear Restricted Four Body Problem

EoM

Let the positions of the primaries be fixed at $x = x_1, x_2, x_3$ on the x-axis in a rotating coordinate system. Let (x, y) denote the coordinates of the infinitesimal particle.

The EoM are given by Idrisi et al. (2025):

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} &= \Theta_x \\ \ddot{y} + 2n\dot{x} &= \Theta_y \end{aligned}\right\} \tag{7}$$

The potential function $\Theta(x, y)$ can be expressed as:

$$\Theta(x, y) = \frac{n^2}{2}(x^2 + y^2) + \frac{\mu_0}{r_0} + \frac{1-\mu-\mu_0}{r_1} + \frac{\mu}{r_2}. \tag{8}$$

The system admits a conserved quantity known as the **Jacobi integral**, given by:

$$C = 2\Theta(x, y) - (\dot{x}^2 + \dot{y}^2) \tag{9}$$

This integral plays a crucial role in determining the allowable regions of motion (zero-velocity curves).

Special Cases

- If the central mass tends to zero, the system reduces to the CR3BP.
- Symmetric mass configurations simplify the analysis and may lead to analytical approximations of equilibrium points.

1.3 Photo-gravitational Effect

Small particles in realistic celestial systems are impacted by pressure of radiation from the sources of light like stars in addition to gravitational forces. The radiating body's effective gravitational attraction has decreased via radiation pressure. Furthermore, a portion of this radiation may be reflected by adjacent bodies, creating albedo or reflected radiation effects that further affect the motion of particles like dust grains, meteoroids, and light-sensitive spacecraft.

1.4 Stability Analysis

An essential component of celestial mechanics is the equilibrium point's stability. These points show locations in the rotating frame where the net force acting on the particle is zero. The direct and reflected radiation forces modifies the resultant potential function and may significantly modify both the location and dynamical stability of these points. Studying their stability helps to understand the motion of particles in complex multi-body environments.

2. Literature Review

Euler (1767) introduced the concept of restricted problem in connection with his lunar theories. R3BP can be defined as a basic model used in Celestial Mechanics to describe the dynamics of a third infinitesimal mass particle due to the gravitational attraction of two heavier bodies called primaries. The three primary bodies orbit in circular motion about the shared center of mass in the CR3BP and the infinitesimal body does not have any influence on the motion and positions of the three primaries. In this effective potential function control the motion of particle and both Coriolis and Centrifugal forces are introduced. Lagrange (1772) explored the five equilibrium points of a three-body system. Chernikov (1970) discussed the sun-planet-particle configuration and showed that the equations of motion have six libration points without Jacobi Integral. An extension of the CR3BP is the R4BP. In the collinear restricted four body problem, three massive bodies attract each other via gravity, and the particle with negligible mass evolves under their influence. Simó (1978) investigated relative equilibrium solutions in multi-body gravitational systems and for four body problem provide the totality of solution when all the four masses are equal. Schuerman (1980) investigated CR3BP by incorporating radiation pressure and the Poynting–Robertson drag. In this the radiation effect was characterized by a parameter representing the ratio of radiation force to gravitational force, which effectively reduces the gravitational attraction of the radiating primary. This study also demonstrated that radiation modifies the potential function and shifts the equilibrium point's locations. Furthermore, author discussed the inclusion of the Poynting–Robertson drag leading to the instability of triangular equilibrium points over long time scales. Michalodimitrakis (1981) presented work on the R4BP. Simmons et al. (1985) discussed the effects of a radiation pressure force on the mass of a test particle due to the large masses, which are both arbitrarily luminous. Kunitsyn & Tureshbaev (1985) studied the motion of R3BP with photo gravitational effect. Markellos et.al (1996) examined the CR3BP with oblateness. Murray & Dermott (1999) discussed the existence of equilibrium (libration) points, where the net force which acts on the infinitesimal particle becomes zero. The positions of equilibrium points are obtained from the stationary conditions of the effective potential. Kalvouridis et al. (2006) discussed the dynamical behavior of the small particle which is subjected to both the gravitational attraction and the radiation pressure of the three heavy primaries is studied and some features of this model, like the zero-velocity curves and surfaces, as well as the equilibrium positions and their stability. Papadouris & Papadakis (2012) investigated equilibrium points of the photo-gravitational

R4BP and demonstrated that radiation pressure strongly influences equilibrium point locations.(Papadouris & Papadakis, 2014) studied solutions in the photogravitational R4BP and results showed that radiation pressure alter the stability and existence of periodic orbits. (Singh & Vincent, 2015) analyzed equilibrium points which are out-of-plane in the photogravitational R4BP and demonstrated that radiation pressure significantly alters system stability.(Arribas et al., 2016) examined equilibrium points of the restricted four-body system which has symmetrical collinear configuration problem and showed that stability attributes and location of equilibrium points are greatly altered by the pressure of radiation. Osorio-Vargas et al. (2020) investigated the orbital dynamics of the photogravitational R4BP and analyzed the influence of radiation pressure on particle trajectories. Suraj et al. (2020) examined convergence of basins in the R4BP and discussed that radiation pressure significantly alters convergence regions associated with equilibrium points. Suraj et al. (2020) analyzed the linear stability of stationary points and their numerical results showed that axis-aligned libration points may become linearly stable for certain values of mass and oblateness parameters, whereas the non-collinear points remain unstable. They also investigated regions of possible motion and basins of convergence using the Newton–Raphson iterative method. Muhammad et al. (2020) analyzed how the triaxiality parameters influence the number, locations and linear stability of coplanar libration points using numerical methods. Suraj et al. (2024) discussed the existence and number of collinear and non-collinear equilibrium points depend on the mass and shape parameter of the primaries**,** with an application to the Saturn–Moon–Moon system**.**In addition to direct radiation pressure, reflected radiation or albedo effects can also influence particle motion in celestial systems.Kumar et al. (2024) analysed the existence, position, and linear stability of nine equilibrium points (three collinear and six non-collinear)**,** examined the motion around them and some llinear points remain stable while while most non-collinear points exhibit stability**,** and the albedo and disc parameters significantly influence Jacobi's constant and the regions of possible motion. Alrebdi et al. (2024)investigated equilibrium points in the collinear restricted four-body problem considering additional perturbations. Their results highlight the importance of studying more generalized dynamical systems that include radiation effects and other perturbations. Chauhan et al. (2025) investigated a perturbed photo- gravitational restricted three-body problem including oblateness, albedo effect, and a straight-segment secondary primary and told that EPs which lie on straight line(x-axis) are always unstable, while non-collinear points may become stable under certain parameter values**,** and the RoM expands as the Jacobi constant

decreases**.** Jangid et al. (2025) discussed that radiation pressure has the strongest influence on EPs and points which are collinear remain unstable while non-collinear points can be stable, and these perturbations also affect the orbits which are periodic around the equilibrium points. Idrisi et al.(2025) extended the classical R3BP to R4BP with collinear configuration and a dominant central primary to model real planetary systems and also analyzed the existence, bifurcation, and linear stability of six libration points, showing that the points which lie on straight line points are mostly unstable while non-collinear points remain stable within a certain parameter range. The model was further applied to the Saturn–Janus–Epimetheus system to demonstrate its astrophysical relevance.

3. Research Gap

Although extensive work has been conducted on the restricted three-body and restricted four-body problems, most studies have focused primarily on gravitational interactions or direct radiation pressure effects. The influence of reflected radiation or albedo effects in the collinear restricted four-body problem has received relatively limited attention. Furthermore, the effect of direct and reflected radiation on the motion of an infinitesimal mass particle in CR4BP remain insufficiently explored. Therefore, the present research aims to investigate the dynamics of an infinitesimal mass particle under the combined influence of direct and reflected radiation in the collinear restricted four-body problem.

4. Significance of the study

The study of the dynamics of an infinitesimal mass particle under direct and reflected radiations in the collinear restricted four-body problem is significant because it extends the classical models of celestial mechanics by incorporating important non-gravitational effects. Traditional restricted four-body models mainly consider gravitational interactions, whereas radiation forces also affect tiny particles in realistic astrophysical environments. The inclusion of both direct radiation from a luminous dominant primary and reflected radiation from nearby bodies provides a more physically realistic representation of multi-body dynamical systems. This study helps in understanding how these radiation forces alter the effective potential, system's equilibrium configurations, and stability of libration points. The outcomes of this study can contribute to the development of celestial mechanics and provide light how dust particles, meteoroids, and other small bodies move in planetary systems where radiation plays a significant role. Furthermore, understanding these dynamical effects is also relevant for modern space technology, particularly for light-propelled spacecraft like solar sails, where radiation forces significantly influence orbital motion. Thus, the research improves both the theoretical foundation of multi-body dynamical systems and their real-world applications in astrodynamics and space exploration.

5. Problem Statement

To create and examine a modified dynamical model of the collinear restricted four-body problem that includes the effects of both direct and reflected radiations.

6. Objectives of Research

The objectives of the proposed research are as follows:

1. To formulate the EoM for the CR4BP with a dominant central mass under the following configurations:

a) when the central dominant mass acts as a radiating body, meanwhile the other two less massive primaries are non-radiating,

b) when the central dominant mass is radiating and the other two less massive primaries act as radiation-reflecting (albedo) bodies.

2. To derive the analytical conditions for the existence and precise location of equilibrium (libration) points in the presence of:

a) radiation pressure from the dominant primary,

b) reflected radiation effects from the smaller primaries.

3. To investigate the influence of

a) radiation pressure, and

b) reflected radiation (albedo effect)

on the linear stability of the equilibrium points as functions of radiation factors and mass parameters.

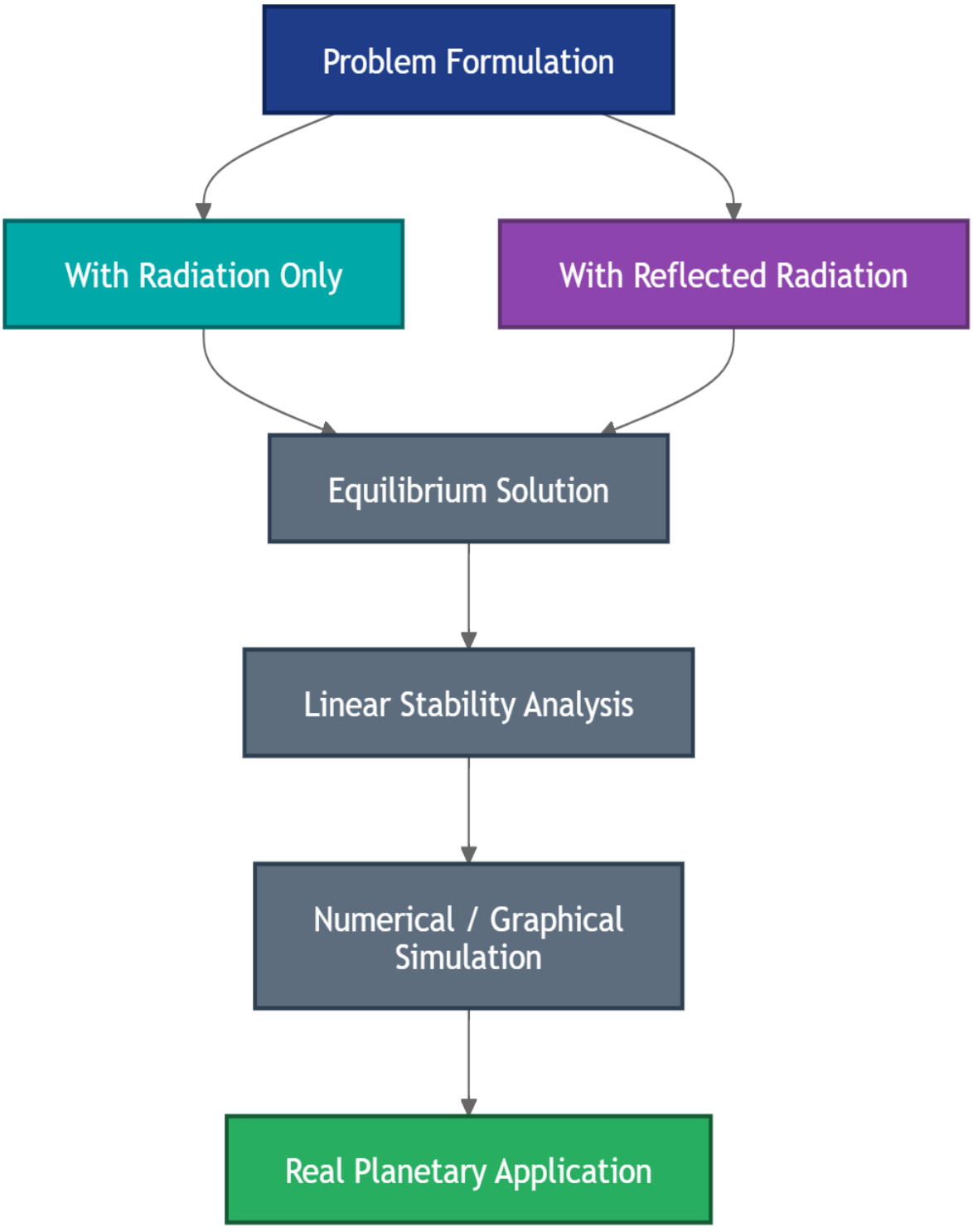
4. To carry out numerical simulations to validate the analytical findings and to compare the dynamical behavior of the system under different radiation and reflection scenarios.

7. Methodology

The methodology of this proposed research will follow a systematic sequence combining theoretical development and computational investigation to examine the influence of direct radiation from the dominant central mass and reflected radiation (albedo effect) from the two smaller primaries on the equilibrium and stability behavior in the generalized framework for the CR4BP with a central dominant mass.

We will follow the following steps for conducting our research

1. Using the potential function as described in Idrisi et.al (2025) , we will develop the equations of motion for infinitesimal mass under the consideration of radiation and reflected radiations effect.
2. Using the conditions of equilibrium solutions, we will explore the collinear and non-collinear equilibrium points analytically, numerically and graphically.
3. After finding the equilibrium points, we will analyze the linear stability of these points.
4. Finally, we will apply our findings to the real planetary system.
5. We will use the software MATHEMATICA for simulations.


Problem Formulation
With Radiation Only
With Reflected Radiation
Equilibrium Solution
Linear Stability Analysis
Numerical / Graphical Simulation
Real Planetary Application

8. Expected Outcomes of the Research

The proposed study is expected to produce the following outcomes:

- Extend the work of Idrisi et al. (2025) to develop a mathematical model for the collinear restricted four-body problem that includes a dominant central mass.
- To determine the stability conditions for the equilibrium points as a function of radiation, albedo and mass parameters.
- Using MATHEMATICA, we will plot zero-velocity curves/regions for the motion of an infinitesimal mass.
- Finally, we will compare our results with the existing literature reviewed.
- We will apply our findings to the real planetary system.

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Researcher: Usha

Registration Number: MRU MPHDN0124210022

Supervisor: Dr. Mohammad Javed Idrisi

Department: Sciences (Mathematics)

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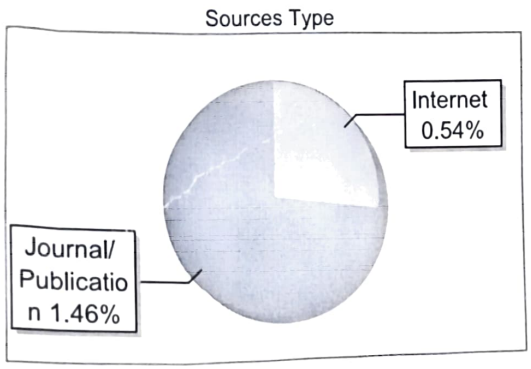

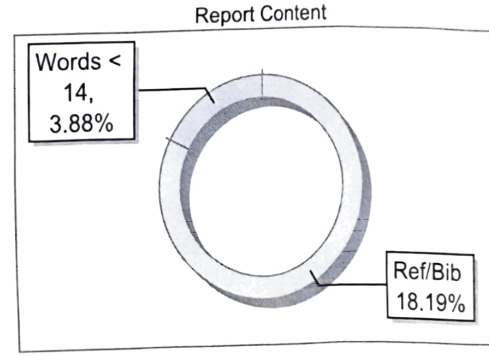

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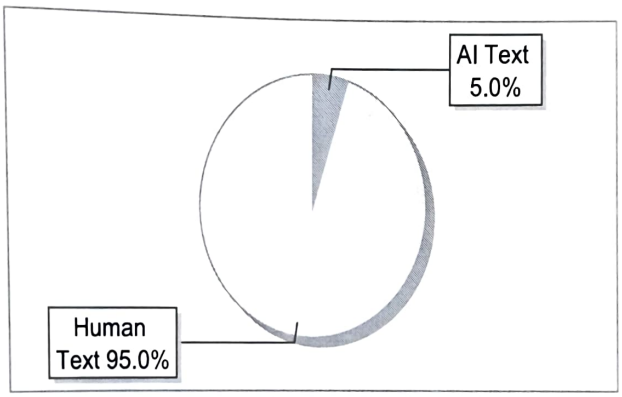

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